Accounting for Earth Curvature in Directional Drilling

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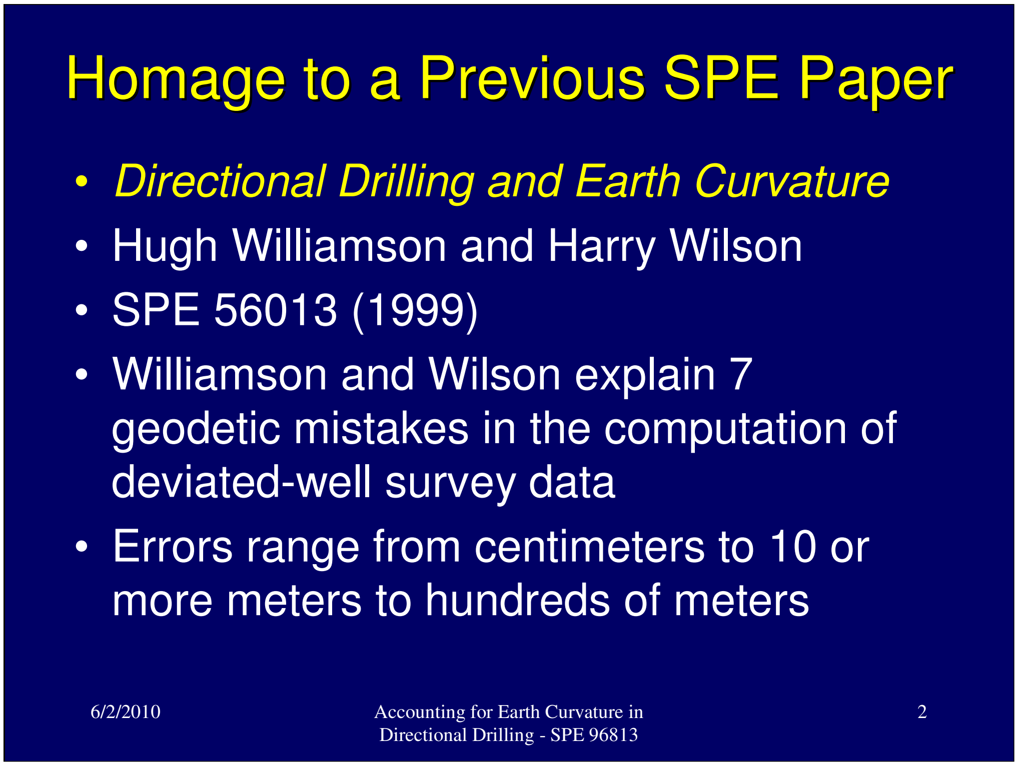


I begin with homage to a previous paper. In an important contribution to the geodesy of directional drilling, authors Hugh Williamson and Harry Wilson discuss the causes - and offer cures - for 7 geodetic mistakes found in current oil field practice. The positional errors resulting from these mistakes range from a few centimeters to about 10 meters to hundreds of meters.

Mistakes of Earth Curvature

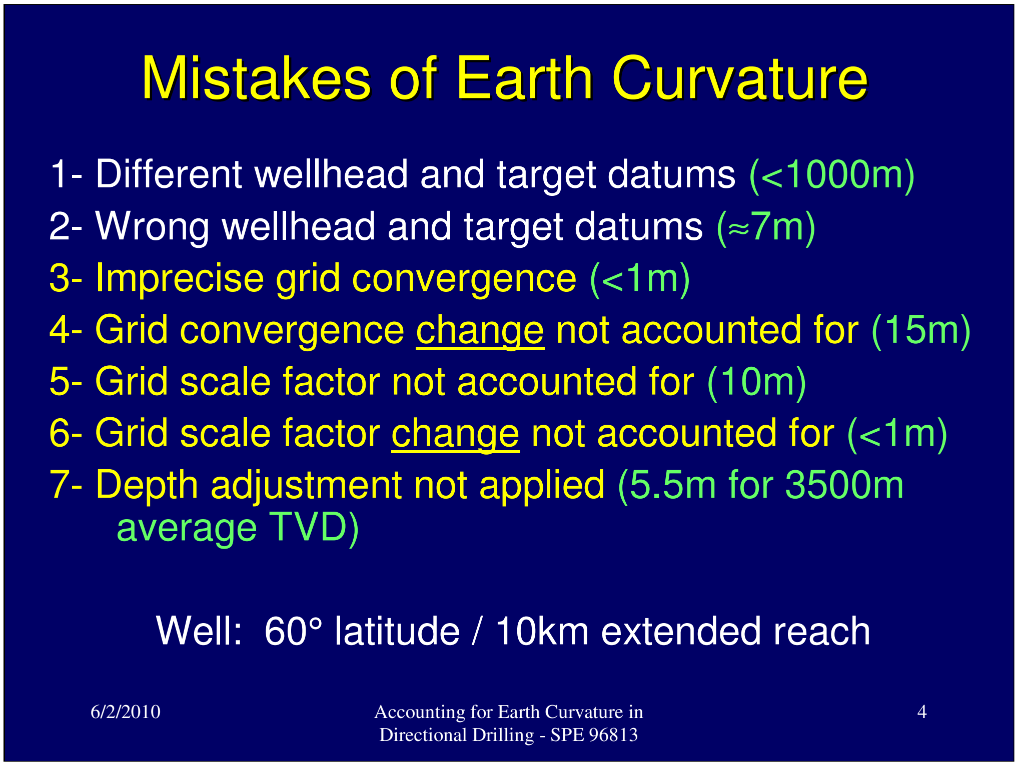
1- Different wellhead and target datums
2- Wrong wellhead and target datums
3- Imprecise grid convergence
4- Grid convergence change not accounted for
5- Grid scale factor not accounted for
6- Grid scale factor change not accounted for
7- Depth adjustment not applied

Of the 7 mistakes cited by Williamson and Wilson, the 2 mistakes shown in white at the top are lapses in geodetic planning that must be prevented by prudent practice no matter how the survey is executed or computed.

The 5 five mistakes in yellow are of a different character. These are not planning lapses or unavoidable measurement errors. They are computational mistakes that can be corrected by changes in software.

Numbers 3, 4, 5 and 6 relate to the use of map projections to convert displacements (also known as departures) into geodetic coordinates.

Mistakes number 4 and number 7 are common in the software I've tested. These mistakes are not accounting for changes in convergence over the reach of a well and not adjusting for depth.



The 7 mistakes are detailed here with quantified positioning errors shown to the right for the extreme well detailed at the bottom of the slide. The well is high-latitude and extended in reach. The seventh mistake is quantified for a very deep well. These extreme features are within the scope of today's technology.

Don't be concerned about the kilometer error shown on the first line. It won't happen to you and it's not the subject of this paper.

The Solutions

- Williamson and Wilson offer depth amplification and map-based applications of grid-north convergence and linear scale factor as the cure for 5 of the 7 mistakes
- This paper offers a simple alternative (LMP) that avoids grid north, requires no mapping corrections, is computationally efficient and equally accurate

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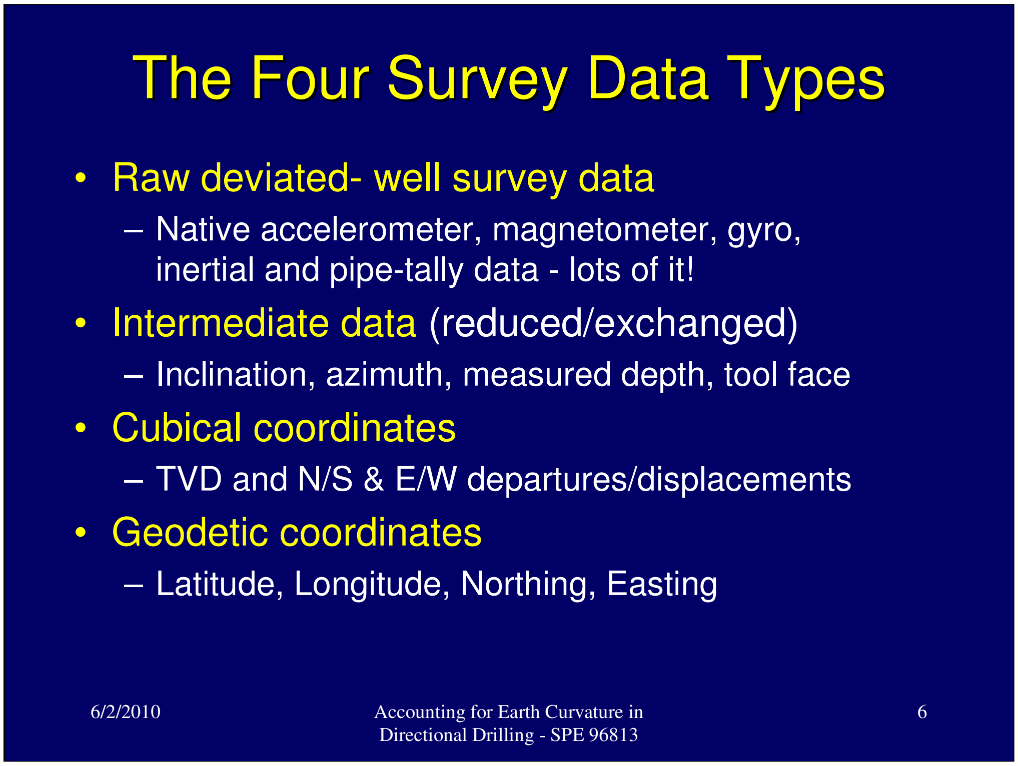
Williamson and Wilson offer the application of depth amplification and map-based corrections for convergence of the meridians and linear scale factor as the solution for the 5 computational mistakes.

In this paper I take a radically different approach. I offer an alternative algorithm (called LMP) that converts True Vertical Depth and horizontal displacements directly into latitude and longitude without an intervening map projection, without convergence of the meridians and without linear scale factors. Instead, LMP uses three easily computed radii of the ellipsoidal Earth. Repeated computation of these radii is more efficient than computing map corrections. Grid north is avoided, and I'll have much to say about grid north in this talk.



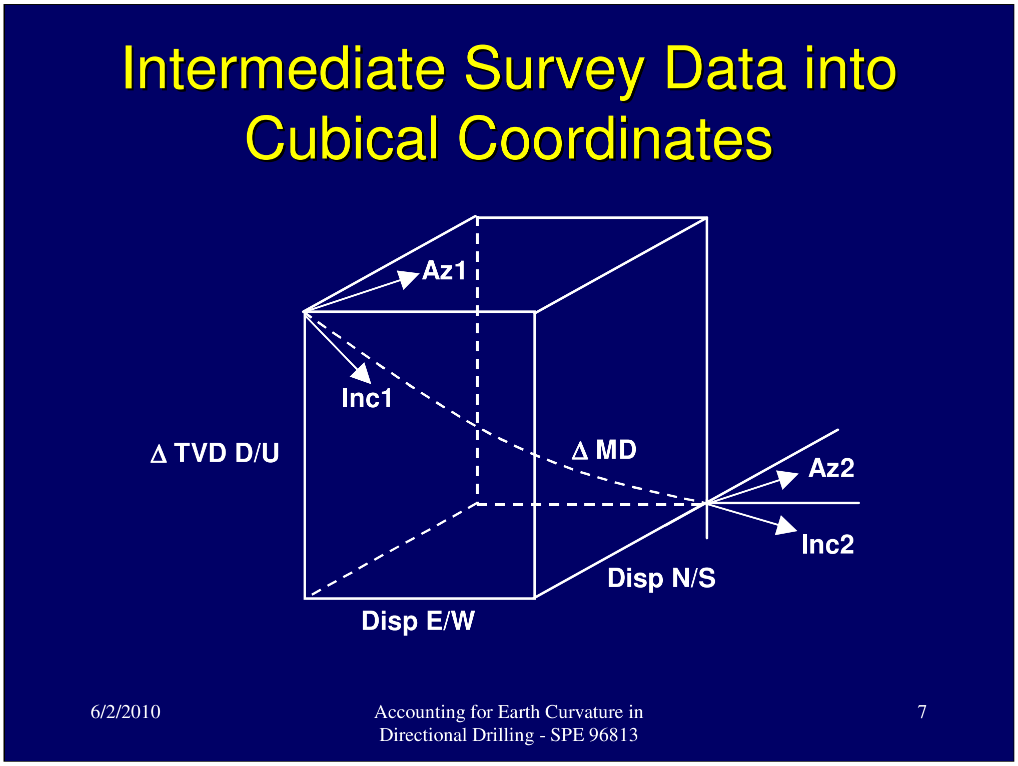
To better understand the problems that LMP solves, I begin with the following taxonomy of survey data types and the formats in which these data are delivered.

Raw survey data are those acquired by field instruments – accelerometers, magnetometers, gyros, and – of course – pipe tally. These observations are acquired in great quantity and are rarely seen in the operating companies.

Instead, raw data are reduced into intermediate data (inclination, azimuth, measured depth). Intermediate data is delivered as the permanent record of a well survey.

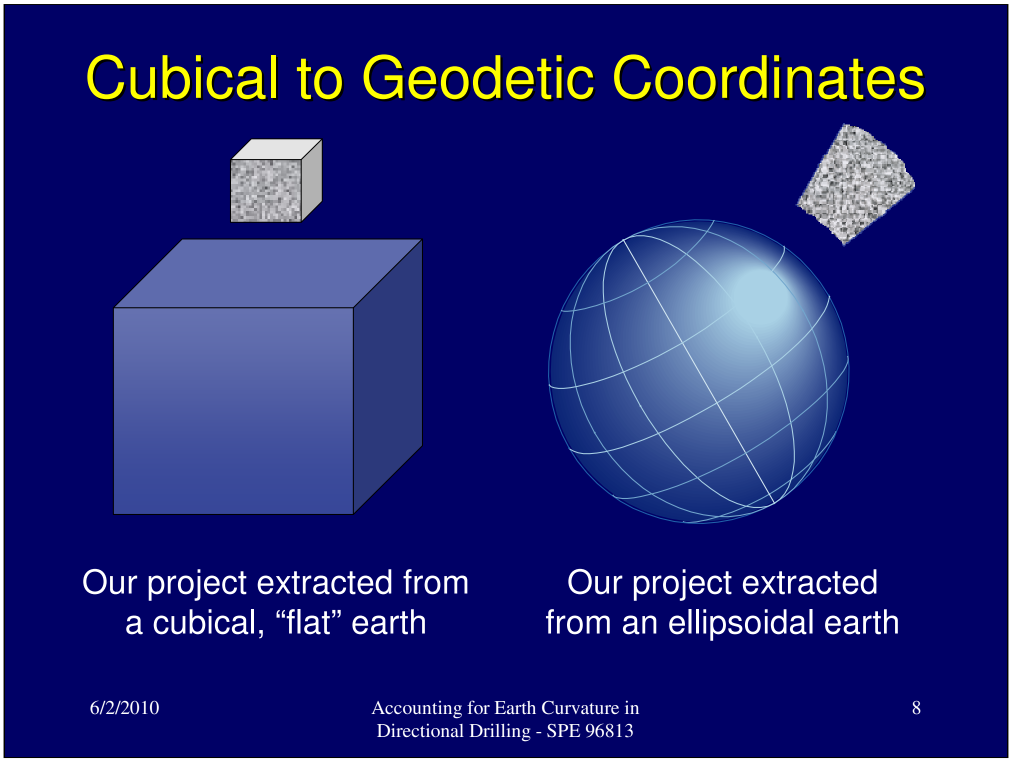
Cubical – or engineering - coordinates are True Vertical Depth and displacements in the N/S and E/W directions.

Geodetic coordinates in the horizontal are either geographical coordinates (latitude and longitude) or projected coordinates (Northings and Eastings, or X's and Y's). TVD remains unchanged as the vertical coordinate.



If the first computation is the reduction of raw data into its intermediate form, the second computation is the reduction of intermediate data into cubical coordinates.

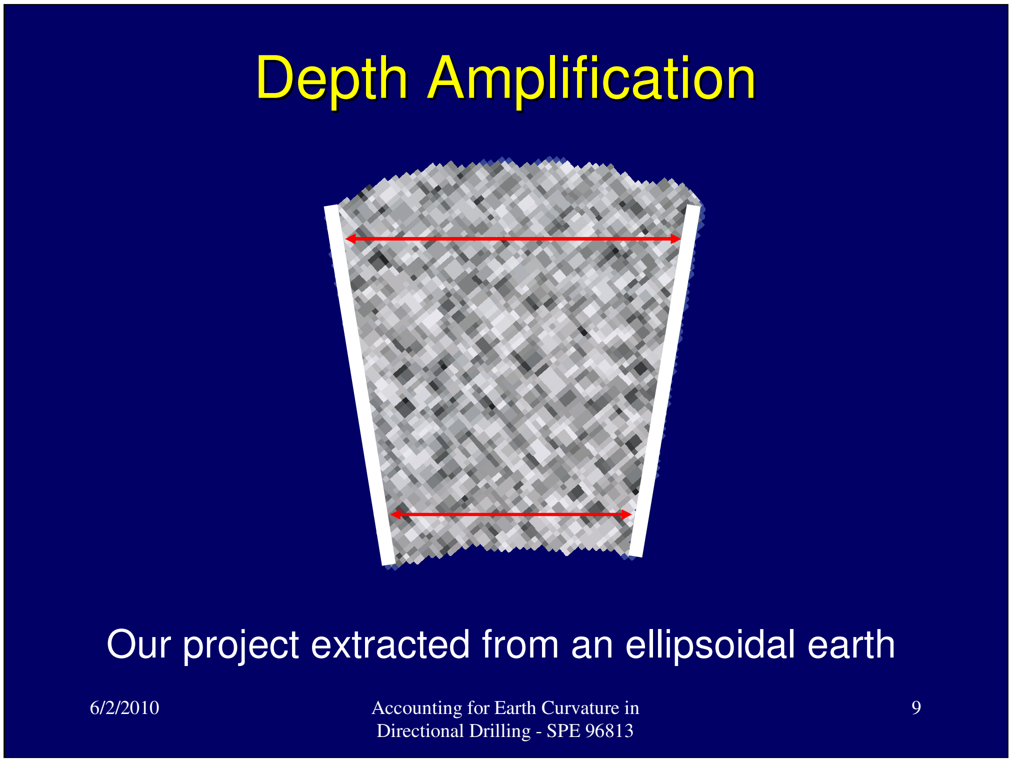
In this graphic our survey stations are at diagonally opposite vertices of the box. The five measurements are the inclination and azimuth entering the box, the inclination and azimuth leaving the box and the difference in measured depth between the two vertices shown as the gentle curve. We must solve for three cubical coordinates: (1) N/S displacement, (2) E/W displacement and (2) delta TVD Down/Up. Simple formulas to accomplish this by different methods (including Minimum Curvature) are given by the API.



Because the Earth is not a cube, cubical coordinates are an inadequate Reference System for important technical and safety issues, such as avoiding well collisions. Cubical coordinates must be converted into geodetic coordinates. The usual way to accomplish this is to relate cubical coordinates to Northings and Eastings determined with a map projection.

This does work, but – as previously mentioned - complete geodetic accuracy requires the computation and application of several corrections at every station along the well path. Unfortunately, these corrections are not always computed and applied. Errors result.

Furthermore, map projections introduce grid north as an azimuth reference in addition to true north and magnetic north. Grid north is a common cause of mistakes in the management of survey data.



If this graphic appears fuzzy to you, think of it as seismic data!

Surveyors using conventional equipment have historically surveyed from hills, buildings or towers for better visibility. These surveyors always made a "height reduction" (or shortening) of the distances observed at these elevations to the surface of the ellipsoid.

Because of Earth curvature, the vertical lines shown in white on either side of the graphic are not parallel (as they are in a cube). They tend to meet at the geocenter. As the elevation changes along two verticals, the horizontal distance between them changes (as shown in red). The geographical coordinates of the verticals don't change, just the distance between them.

Similarly, a displacement at depth changes geographical coordinates faster than the same displacement near the surface.

The seventh mistake cited by Williamson and Wilson is the lack of this correction.

Contractor Survey Formats

- Intermediate data and cubical and geodetic coordinates delivered in a spreadsheet
- Formats specific to survey companies
- Often referenced to grid north
  - – An unfortunate coupling of computational convenience with field acquisition
  - – True or magnetic north can be observed with instruments – grid north cannot
  - – Grid north is a source of data-management confusion

Survey contractors typically deliver intermediate data, cubical coordinates and geodetic coordinates in a spreadsheet in formats that are specific to the specific contractors. Cubical coordinates are not the same as projected coordinates. Some presentations are well documented with respect to this distinction and the differences among true, magnetic and grid north. Some are not.

As mentioned, the use of a map projection in the conversion of cubical to geodetic coordinates necessarily introduces grid north as an azimuth reference. Unlike true or magnetic north, which can be physically observed in the field with appropriate instruments, grid north is a mathematical abstraction. Grid north is a consequence of the grid parameters chosen for the projection, either by convention, for convenience or rather arbitrarily. Change the grid parameters, especially the central meridian, and grid north changes.

In the operating companies a regional geological study may, indeed, require a change of grid parameters. Australian map projections reverse the sign convention for convergence! Grid north can also be found in field acquisition, for example, free gyros aligned with a platform grid-north reference. But the interpretation project using the data may be in a different projection altogether.

These are just some reasons why this confusing coupling of computational convenience with acquisition and exchange can cause mistakes in the management of survey data. Grid north is best avoided - if possible.

Data Format Standards

- UK Offshore Operators Association
  - UKOOA P7/2000: ASCII, intermediate and geodetic data, geodetically unambiguous.
- Minerals Management Service
  - MMS P7/2000: required for US Federal OCS
  - Clone of UKOOA P7/2000
- POSC
  - WITSML: XML raw acquisition data
  - WellPathML: XML version of UKOOA P7 for intermediate and geodetic data

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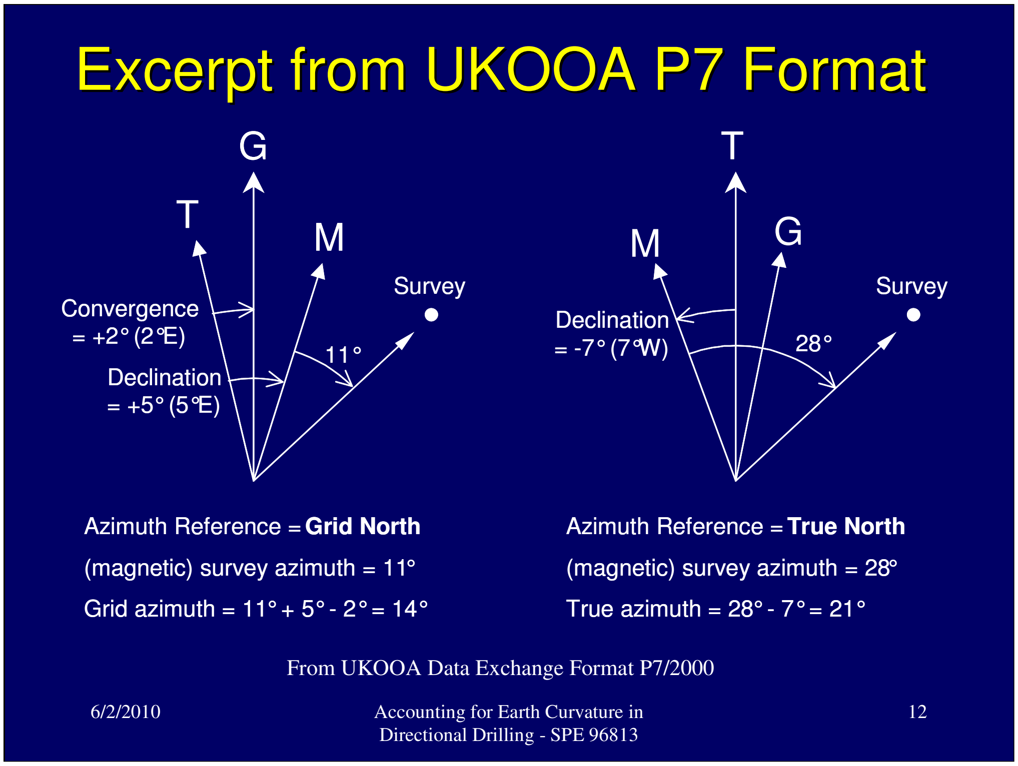
The UK Offshore Operators Association (UKOOA) has addressed the issue of inadequate documentation with the UKOOA P7/2000 format. The P7 format is geodetically unambiguous. It is widely used in Europe.

In the United States the Minerals Management Service (MMS) now requires a clone of UKOOA P7/2000 (called MMS P7/2000) for wells spud in the Federal Outer Continental Shelf after July 26th of last year (2004).

POSC (the Petrotechnical Open Standards Consortium) has two formats.

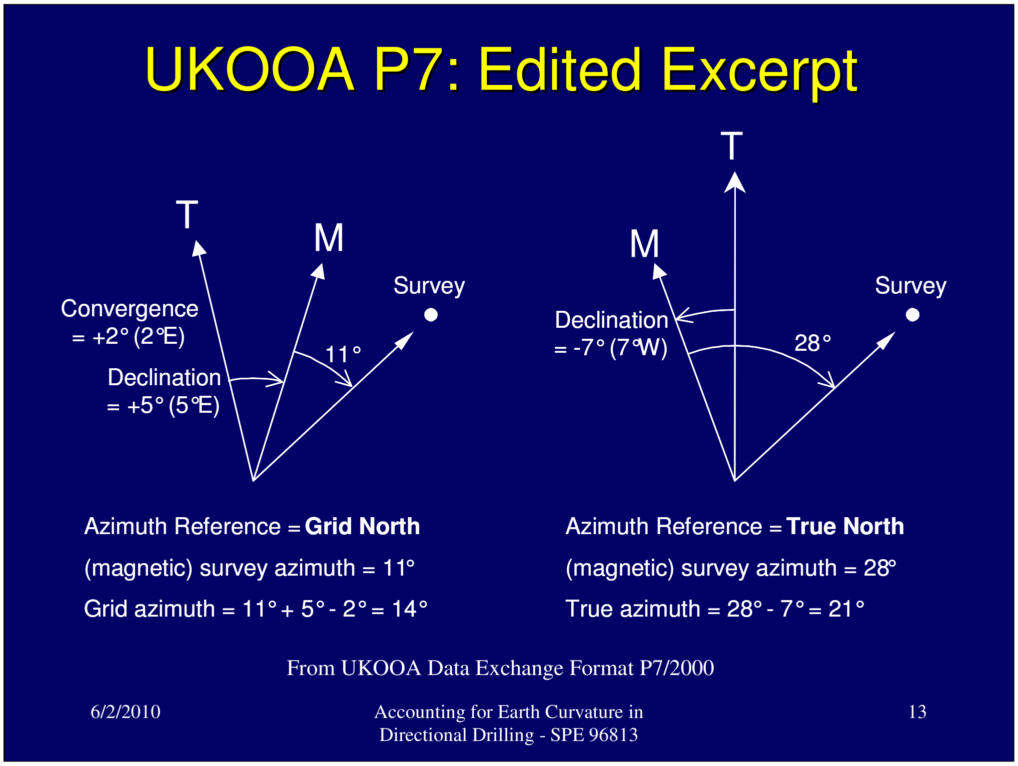
WITSML (Wellsite Information Transfer Standard Markup Language) can capture raw survey data for exchange.

WellPathML is emerging as an XML version of UKOOA P7 for intermediate, cubical and geodetic data.



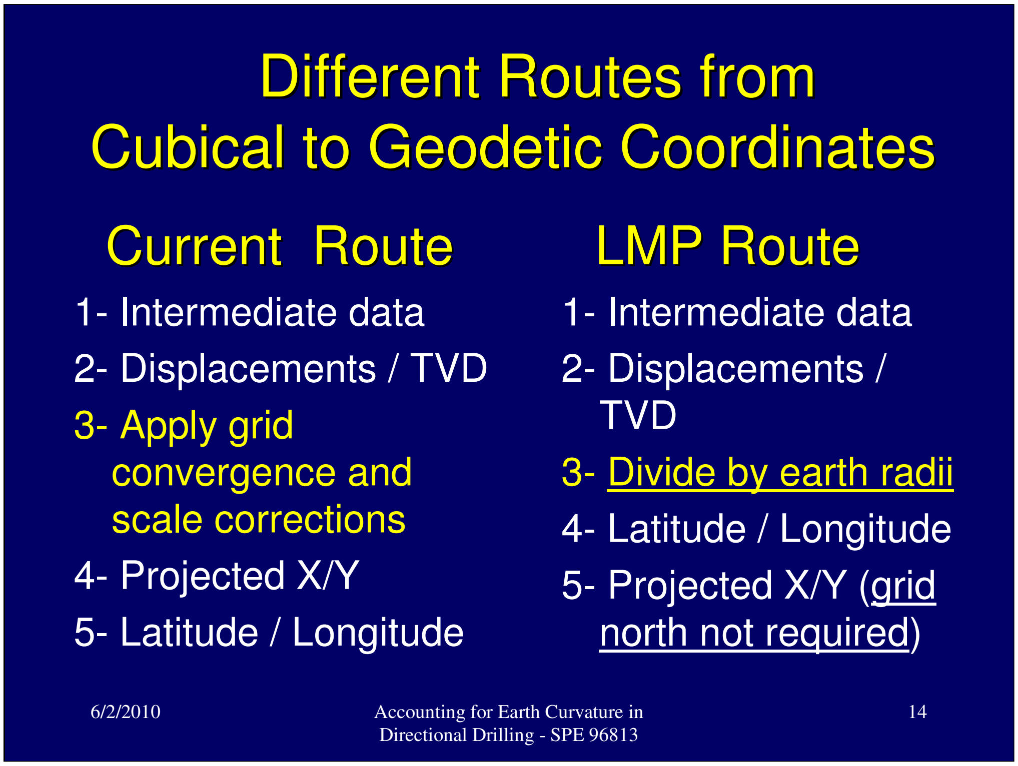
One benefit of the UKOOA P7 format and its inheritors is the clarity of geodetic documentation.

For example, this graphic from the P7 format offers guidance in reconciling true north, magnetic north, grid north and survey direction.



Look how simple this graphic would be if we eliminate grid north from the computations!

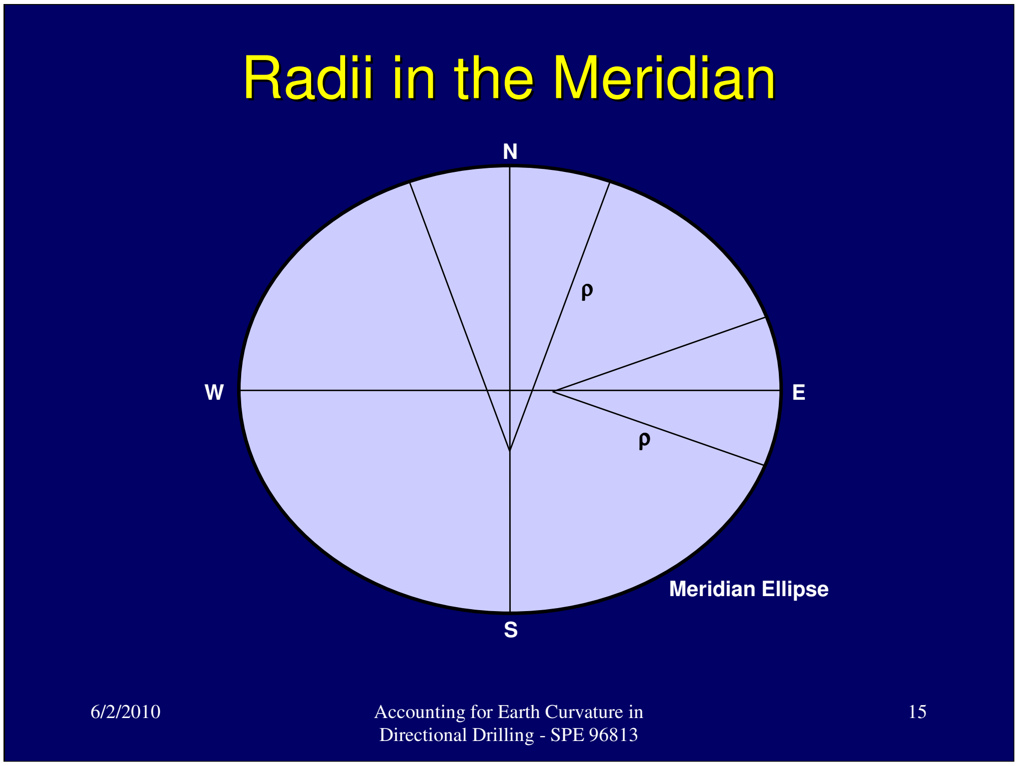
It can be done.



Considering some of the issues discussed so far, an ExxonMobil colleague (David M. Lee) proposed that well-path latitude and longitude be computed directly from cubical coordinates and the radius of the Earth, that is, without a map projection. I modified David's proposal for an ellipsoidal Earth with an infinite number of radii of curvature that vary as functions of latitude and azimuth. The result is Lee's Modified Proposal, or LMP.

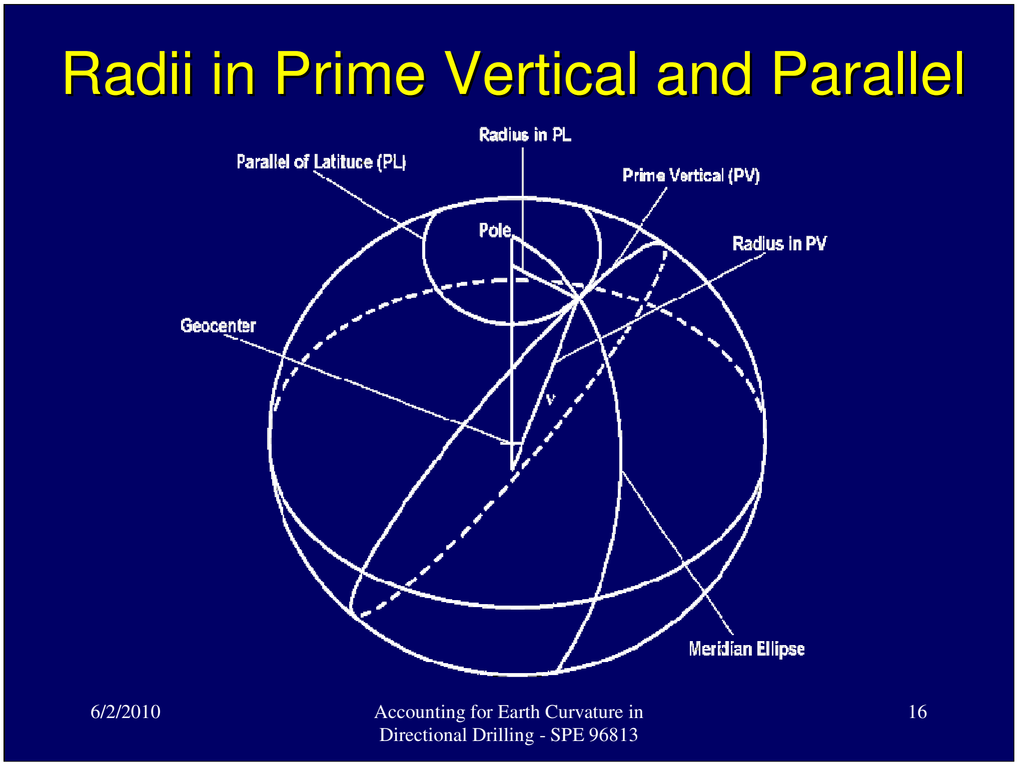
The current route to geodetic coordinates is that on the left. All the map corrections in step 3 are not always applied.

Step 3 in the LMP route to the right of slide is to divide cubical displacements by radii of curvature. The results are differences in latitude and longitude, which are then summed. If projected coordinates are required, latitude and longitude can be converted into Northings and Eastings as is normally done, that is, with forward projection formulas. Deferring this conversion until needed is an advantage in the management of directional survey data.



So, what are these radii of the curved Earth?

This slide exhibits the varying radius of curvature in the meridian arc, which is the N/S ellipse of constant longitude. The radius of curvature in the meridian is called "rho". Its length varies as a function of latitude.



The E/W direction is somewhat more complicated.

The prime vertical is the great circle in the E/W direction. It crosses the Equator. The radius of curvature in the prime vertical is called "nu" (and it's shown here).

The parallel is the small circle of constant latitude in the E/W direction. The radius of curvature in the parallel of latitude is related to the radius of curvature in the prime vertical by the cosine of latitude.

Intermediate Quantities

- Given the semi-major axis (a) and reciprocal of flattening (rf) of the ellipsoid of the datum of the wellhead location
- Then the semi-minor axis (b) is:

$$b = a/(1 - 1/rf)$$

- And the eccentricity squared (e^2) is:

$$e^2 = (a^2 - b^2)/a^2$$

This slide and the next exhibit the formulas for the radii of Earth curvature, which we'll touch on only briefly since they are documented in the paper.

Every well has a surface location in latitude and longitude. Latitude and longitude are referenced to a geodetic datum. Every datum is associated with an ellipsoid. Every ellipsoid has a semi-major axis ("a") and a reciprocal of flattening ("rf"). Given these quantities we can compute two other intermediate quantities, the semi-minor axis ("b") and the eccentricity squared of the ellipsoid ("e-squared").

Radii of Earth Curvature

- Radius of curvature in the meridian (ρ) is:

$$\rho = a \cdot (1 - e^2)/(1 - e^2 \sin^2 \phi)^{3/2}$$

- The prime vertical radius (ν) is:

$$\nu = a/(1 - e^2 \sin^2 \phi)^{1/2}$$

- Radius of curvature in the parallel is:

$$\nu \cdot \cos \phi$$

Given the latitude ("phi") at every station along the wellbore, these simple formulas give the three radii of curvature discussed previously, "rho", "nu" and the radius of curvature in the parallel.

LMP in Pseudocode

1. Divide N/S departure by the radius of curvature in the meridian minus TVD
2. Divide E/W departure by the radius of curvature in the prime vertical minus TVD and again by the cosine of latitude (ϕ)
3. Results are changes in latitude and longitude in radians respectively

So, how are these concepts implemented? LMP is expressed in pseudocode as follows:

<u>First</u>, divide the N/S displacement by the computed radius of curvature in the meridian minus TVD. Subtracting TVD from the Earth radius accomplishes depth amplification. TVD is assumed to be referenced to the geoid, if not to the ellipsoid.

<u>Second</u>, divide the E/W displacement by the computed radius of curvature in the prime vertical minus TVD and again by the cosine of geodetic latitude.

<u>Finally</u>, these results are changes in latitude and longitude respectively expressed in radians.

And we're done!

LMP in Matlab

```
rad=180/pi;                                      % Degrees in a radian
A=6378249.145;                                   % A is semi-major axis
RF=293.465;                                      % RF is reciprocal of flattening
B=A*(1-1/RF);                                    % B is semi-minor axis
E2=1-B^2/A^2;                                    % E2 is eccentricity squared

load data.dat;                                   % Load the data
len=length(data);                                % Count number of survey stations
Edeplast=data(1,6);      Ndeplast=data(1,5);     % Grab the departures
latlast=data(1,7);       lonlast=data(1,8);      % Grab the surface geographicals
for dex=1:len                                    % Cycle through all the stations
   LATRAD=latlast/rad;                           % Degrees to radians
   R=A*(1-E2)/(sqrt(1-E2*sin(LATRAD)^2)^3);      % Radius in the meridian
   N=A/sqrt(1-E2*sin(LATRAD)^2);                 % Radius in the prime vertical
   TVD=data(dex,4);                              % Grab the TVD
   Edep=data(dex,6);     Ndep=data(dex,5);       % Grab the departures
   dEdep=Edep-Edeplast;  dNdep=Ndep-Ndeplast;    % Difference the departures
   Edeplast=Edep;        Ndeplast=Ndep;          % Save the current departures
   dlat=rad*dNdep/(R-TVD);                       % Delta geographicals with LMP ...
   dlon=rad*dEdep/(N-TVD)/cos(LATRAD);           % ... this is the main idea!
   lat=latlast+dlat;     lon=lonlast+dlon;       % Increment geographicals
   latlast=lat;          lonlast=lon;            % Save geographicals for next loop
   data(dex,9)=lat;      data(dex,10)=lon;       % Store the geographicals in data
end
```

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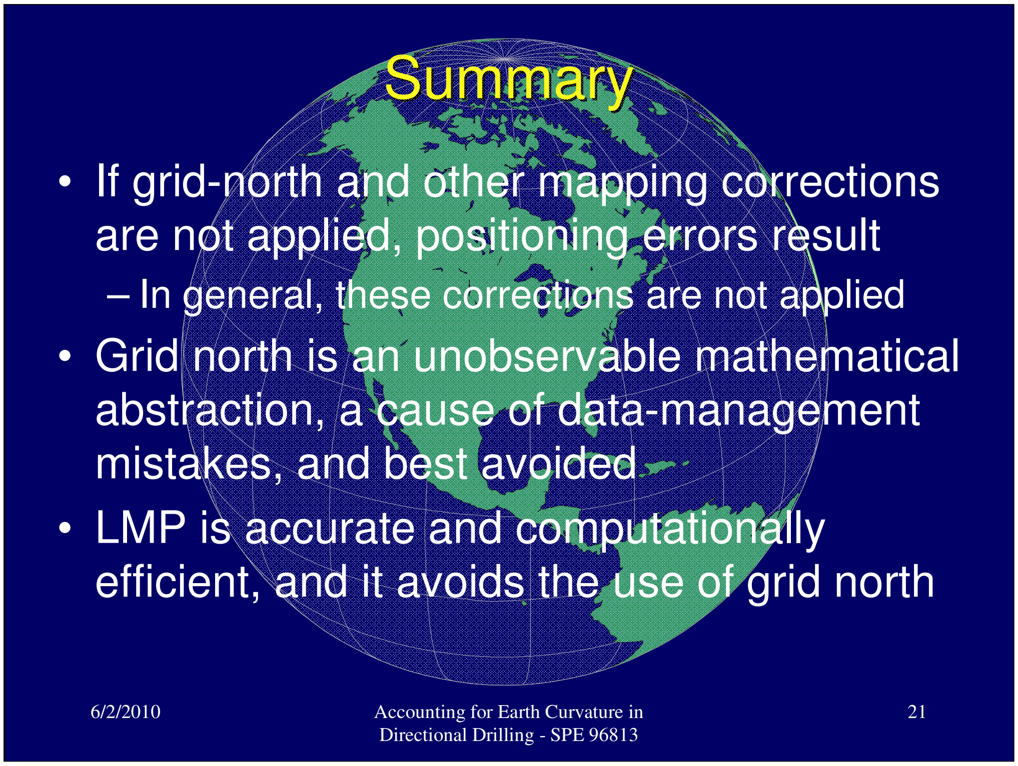
This slide, which exhibits LMP in Matlab code, is also documented in the paper, so we'll not dwell long upon it, either.

The code is on the left. Comments are on the right.

It is worth noting, however, that all the heavy lifting is done by only four lines of code, those highlighted in yellow. Two radii of curvature are computed here. The displacements are divided by the radii minus TVD here.

The rest is bookkeeping.



In summary, LMP is a simple, computationally efficient algorithm that eliminates 5 of the 7 mistakes cited by Williamson and Wilson. The two remaining mistakes pertain to the datum of the surface and target locations. They must be prevented by prudent geodetic practice whether LMP is used or not.

An additional benefit of LMP is that neither grid north nor convergence of the meridians is needed for the computation of geodetic coordinates. This eliminates a common source of mistakes in the management of directional-well survey data.